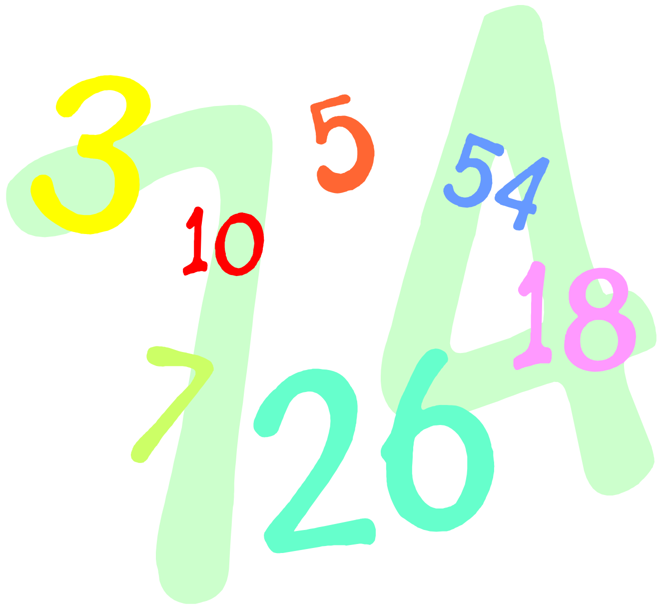
Learning & Growing Together

Fir Tree Junior School

Progression in Calculation Policy



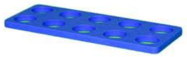
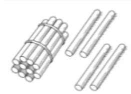
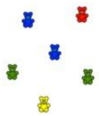
Wallingford Partnership

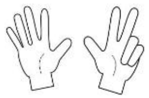
Introduction:

Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use models and images to support their mental and written methods of calculation. As children's mental methods are strengthened and refined they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

<u>First Experiences/Pre-steps:</u>

There are fundamental skills that it is important for children to develop an early understanding of as building blocks to future learning in maths, including that linked to calculation. A selection of the skills include:

- Ordinality – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…)
- Cardinality – 'understanding the value of different numbers' – e.g. (7 =  17 =  +  12 = 
- Equality – 'seven is the same total as four add three' – e.g.  = 
- Subitising – 'instantly recognizing the number of objects in a small group, without counting them' – e.g.  → five
- Conservation of number – 'recognising that a value of objects are the same, even if they are laid out differently' – e.g.  
- One-to-one correspondence – e.g. 

- Counting on and back from any number – e.g. 'five add three more totals eight'  'ten take away three totals seven' 

- Using apparatus and objects to represent and communicate thinking – e.g. 
- Maths language – using mathematical words verbally in every-day situations – e.g. 'climb up to the top' / 'climb down to the bottom'

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice through progression in relevant practical maths experiences and visual representations.

By the end of Year 6, children will be equipped with efficient mental and written calculation methods, which they use with fluency. Decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next Step. At whatever Step in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently.

The overall aims are that when children leave primary school they:

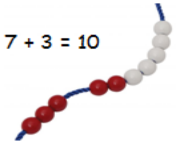
- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value - through experience with practical equipment and visual representations;
- Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

Addition:

Mental Calculation Strategies for Addition and Subtraction

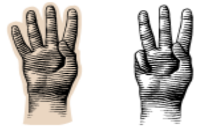
Number Bonds

7 + 3 = 10



Doubles

Near Doubles

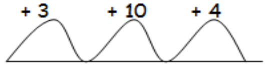


Adjusting

28 + 19 = 47

Bridging

47 + 17 = 64



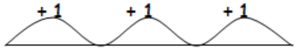
Partitioning

44 + 34 = 78

70 + 8 = 78

Finding the Difference

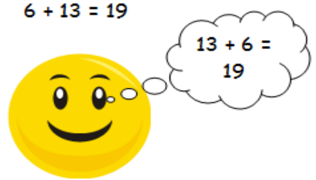
101 - 98 = 3

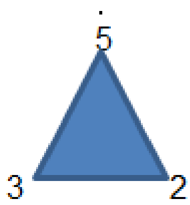
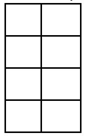
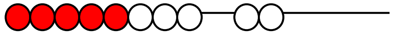
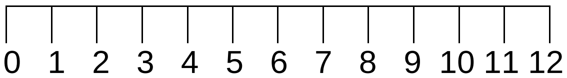
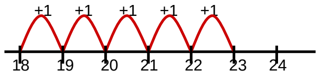
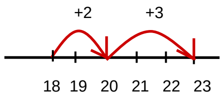


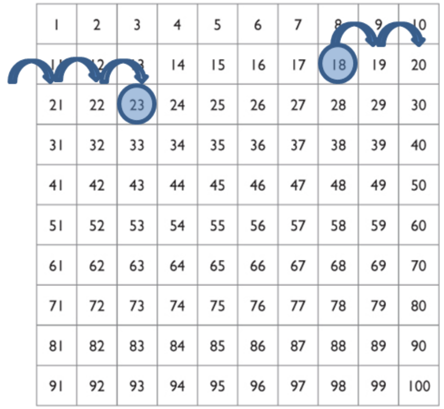
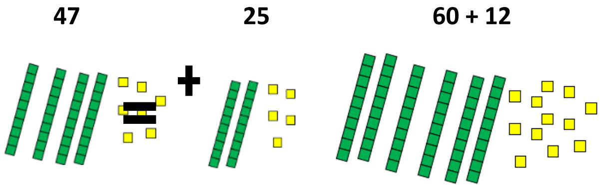
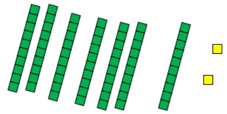
Reordering

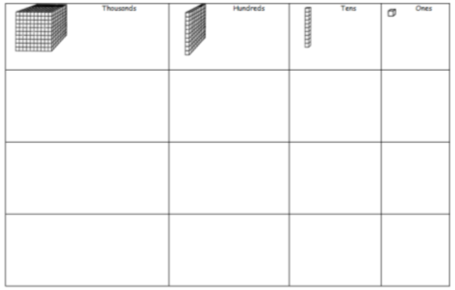
e.g. put big number in head when counting on

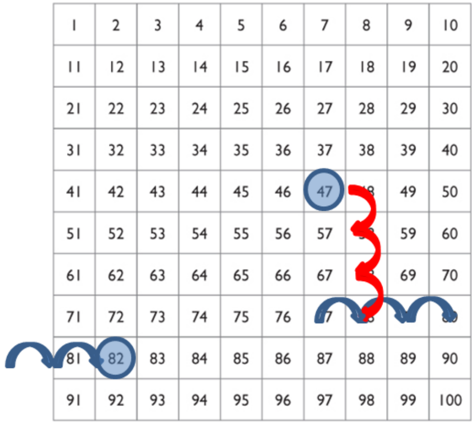
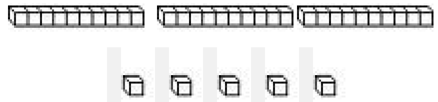
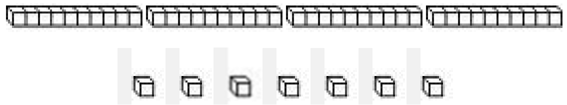
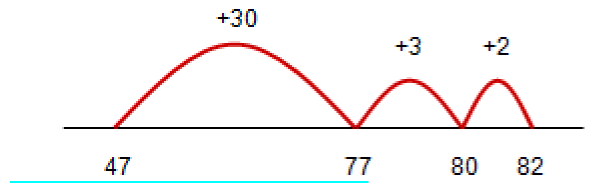
6 + 13 = 19

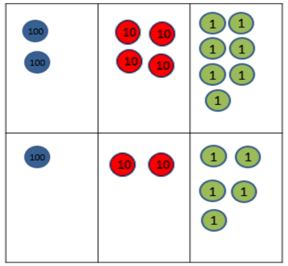
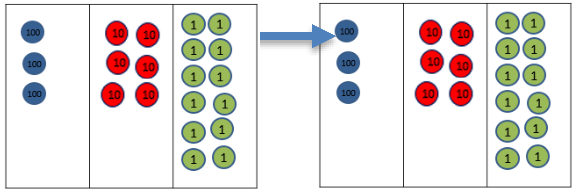


	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Step 1:	Count in ones to and across 100 forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	Pupils use apparatus to explore addition as the inverse of subtraction.   4 add 1 is 5 5 subtract 4 leaves 1	Rapid recall of all pairs of numbers totalling numbers up to 20. Use structured apparatus – i.e. Numicon, tens frames, abaci, etc. 	**Combining two groups:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. • Teachers model use of number tracks to count on or line up counters/objects along the number track. This is a precursor to use of a fully numbered number-line.	  3 + 2 = 5  'eight add two more makes ten'  'one more than four is five' Counting and Combining sets of Objects Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation) 
Step 2:	Continue practicing above skills. Count in steps of 2, 3 and 5 forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as	Reorder numbers when adding, i.e. start with largest number, find bonds, etc. Add doubles and derive near doubles. Round numbers to the nearest 10.	Recall addition facts for all numbers to 20.	**Counting on from any number:** • Children begin to use numbered lines to support their own calculations, initially counting in ones before beginning to work more efficiently. **Counting on from the largest number:** • Children reorder calculations to start with the largest number.	Number line with all numbers labelled  18 + 5  …to… 

	well as a number square.			**Towards a Written Method** Partitioning in different ways and recombine	18+5=  47+25  Leading to exchanging: **72** 

					Thousands Hundreds Tens Ones

Step 3:	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on by 10 or 100 from any two digit number. Link to counting stick: counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Partitioning by bridging through 10 and multiples of 10 when adding. Adjusting when adding 11 or 9 to a number. Relating inverse number operations – using structured apparatus to explore and understand that subtraction undoes addition.	Connect pairs totalling ten to pairs of multiples of 10 totalling 100. Use 10ps in tens frame. Recall pairs of two-digit numbers with a total of 100, i.e. 32 + ? = 100.	**Expanded horizontal addition:** • Add numbers using structured apparatus to support understanding of place value. • Make connections between partitioning both numbers using structured apparatus and partition the second number only using a number line.	 Add…  …and…  By partitioning and recombining 30+ 40 = 70 5 + 7 = 12 70 + 12 = 82 47+35= 

				<u>Towards a Written Method</u> Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)	**<u>Expanded written method</u>** 40 + 7 + 20 + 5 = 40+20 + 7 + 5 = 60 + 12 = 72 40 + 7 + 20 + 5 60 + 12 = 72  247 +125 12 60 300 372 Leading to children understanding the exchange between tens and ones. 

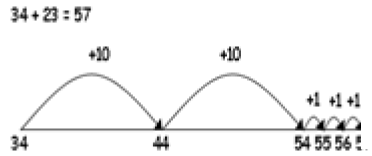
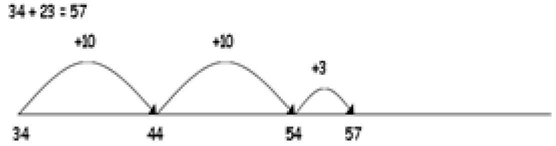
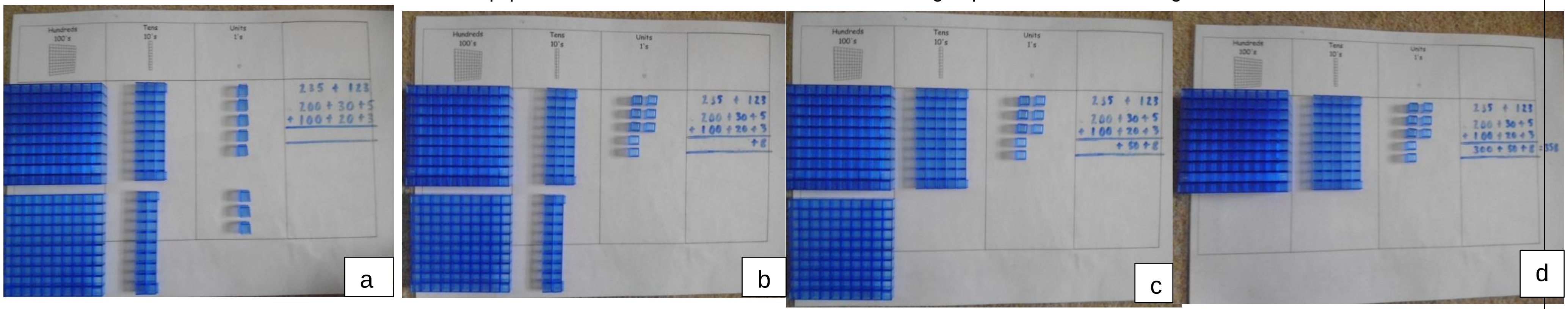
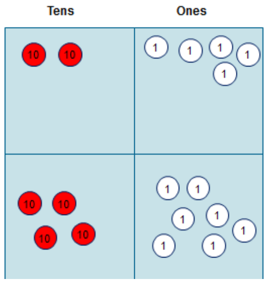
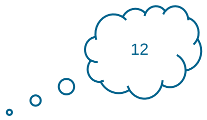
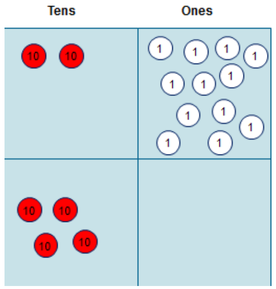
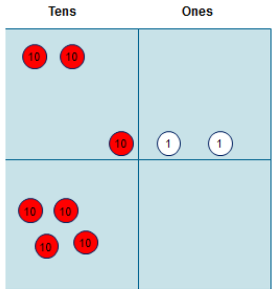
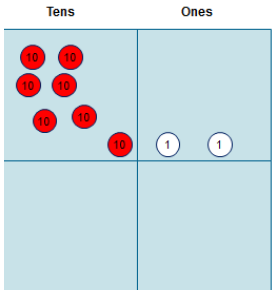
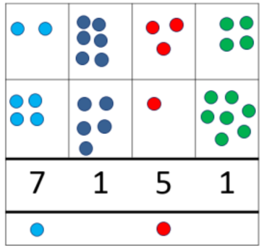
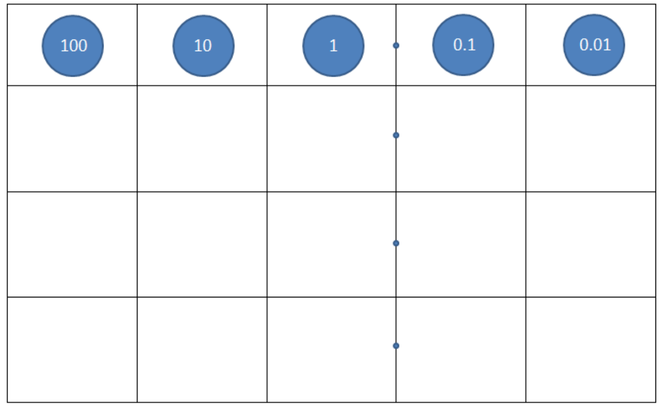
Step 4:	Continue practicing previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes. Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 8 + 3 = 11, I also know 0.8 + 0.3 = 1.1 and 8/100 + 3/100 = 11/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Addition doubles of numbers to 100. Pairs of fractions totalling 1.	**Expanded horizontal method, leading to columnar addition:** • Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete materials, e.g. straws, Numicon, Dienes and place-value cards. • Teachers model how numbers can be partitioned into tens and ones, as well as in different ways, e.g. 20 + 5 10 + 15 • As children move towards using a columnar method, links continue to be made with earlier models and images, including the number line.	It is crucial that empty number lines are kept as well as using more formal written calculation methods. 200 + 40 + 7 100 + 20 + 5 300 + 60 +12 = 372 Counting on in tens an an addition calculation:  247 +125 12 60 300 372 Counting on more efficiently: 

Illustration of how to use Dienes equipment to ensure children have an understanding of place value when using columnar addition.

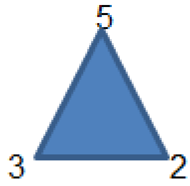
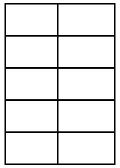
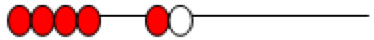
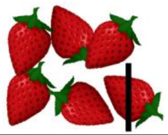


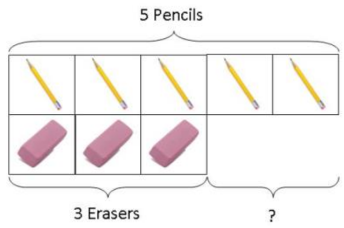
Step 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages. Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Use apparatus and knowledge of place value to add decimals, i.e. 3.8 + 2.5 = 5 + 1.3 Reorder increasingly complex calculations, i.e. 1.7 + 2.8 + 0.3 = 1.7 + 0.3 + 2.8 Rounding and adjusting – i.e. 405 + 399 → add 400 and then subtract 1 Bridging through decimals, i.e. 0.8 + 0.35 = 0.8 + 0.2 + 0.15 using empty number lines. Partitioning using near doubles, i.e. 2.5 + 2.6 = 5 + 0.1 Reorder decimals, i.e. 4.7 + 5.6 – 0.7 …as… 4.7 – 0.7 + 5.6 = 4 + 5.6.	Continue to practice previous Step and make links between known facts and addition pairs for fractions, percentages and decimals Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7 Ensure all children are confident recalling basic facts to 20 and deriving facts using place value. Make links between decimals, fractions and percentages.	**Expanded vertical method, leading to columnar addition:** • Teachers model a column method that records and explains partial mental methods. • There remains an emphasis on the language of calculation, e.g. 'Forty plus seventy equals one-hundred and ten.'… 'Seven add six equals thirteen.' …before recombining numbers. Teachers also model the language of: 'Four tens add seven tens total eleven tens or 110.' Teachers similarly advance to model the addition of two 3-digit numbers with the expectation that as children's knowledge of place value is secured, they become ready to approach a formal compact method. **Columnar addition (formal written method):** • The concept of exchange is introduced through continued use of practical equipment (manipulatives). • Teachers model: 1. "I have two tens and five ones, which need adding to four tens and seven ones." 2. "I add five ones to seven ones, which gives me twelve ones." 3. "I exchange ten of my twelve ones for a ten counter." 4. "I add my three tens and four tens to make seven tens." "Altogether, I have seven tens and two ones."	Informal columnar: Adding the tens first: 47 + 76 110 13 123 Adding the ones first: 47 + 76 13 110 123 **Pupils to be encouraged to consider mental strategies first.** Formal columnar: 25 +47  25 +47 2 1 

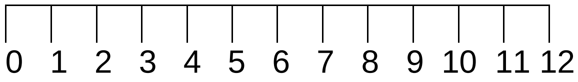
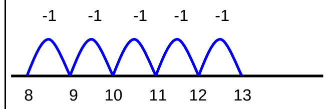
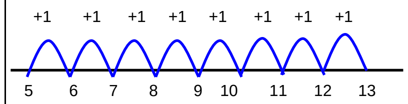
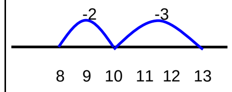
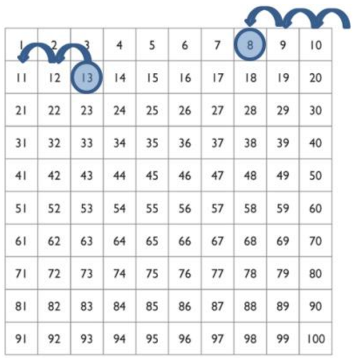
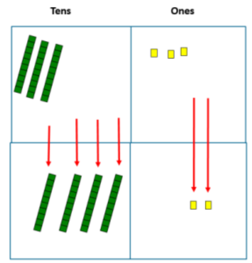
Step 6:				-- • Teachers similarly advance to model the addition of two 3-digit numbers, e.g.	 25 +47 2  +47 72 1  587 + 475 1062 1 1

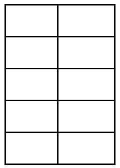
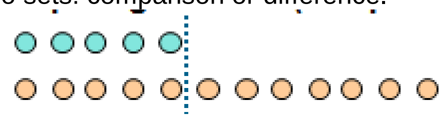
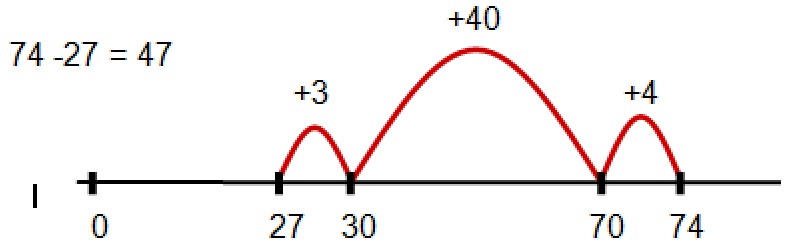
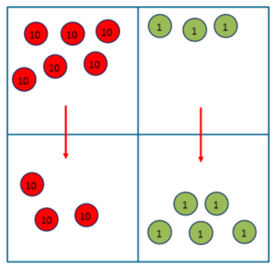
				587 + 475 1062 1 1 • Advance to adding numbers with decimal places • Progress to written column method.	**<u>Compact written method</u>** Extend to numbers with at least four digits.  2634 +4517 7151 1 1  25.14 <u>13.12+</u> 38.26

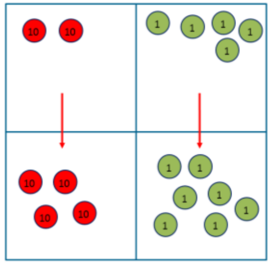
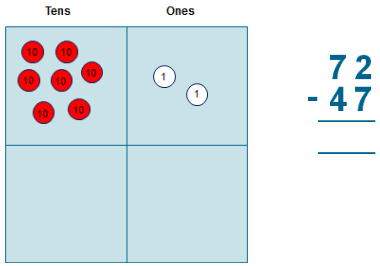
Subtraction:

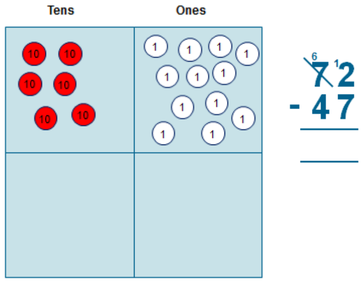
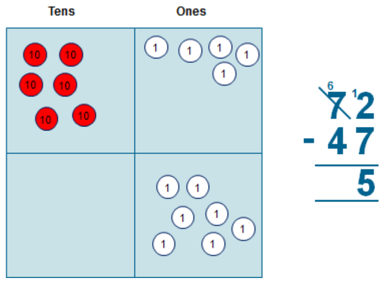
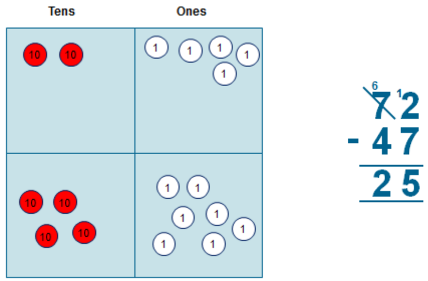
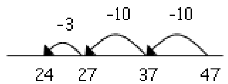
	Counting	Mental strategies	Rapid Recall	Written calculation and appropriate models and images to support conceptual understanding	
Step 1:	Count in ones to and across 100, forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	Pupils use apparatus to explore addition as the inverse of subtraction: .   'four add one is five.' 'five subtract four leaves one'	Rapid recall of subtraction facts for numbers up to 10. Use structured apparatus, i.e. Numicon, tens frames, abaci etc. 	**Subtraction as taking away from a group:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. • Teachers model use of number tracks to count back or remove counters/objects from the number track or set. This is a precursor to use of a fully numbered number-line.	  5 - 2 = 3  'six take away two leaves four'  'one less than six is five' Understand subtraction as 6-1 in a variety of context. 

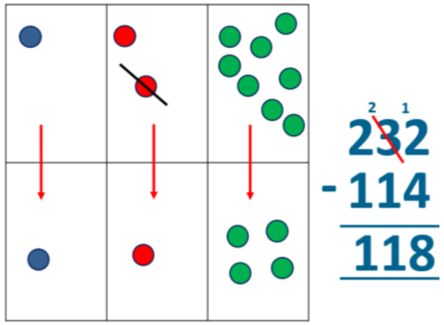
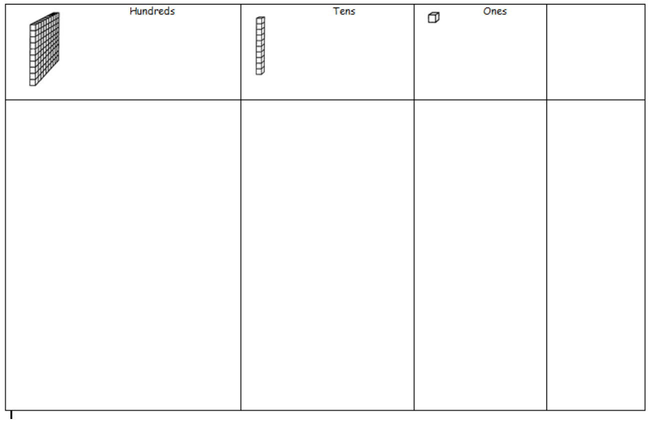
					Introduce children to the Bar Model: 

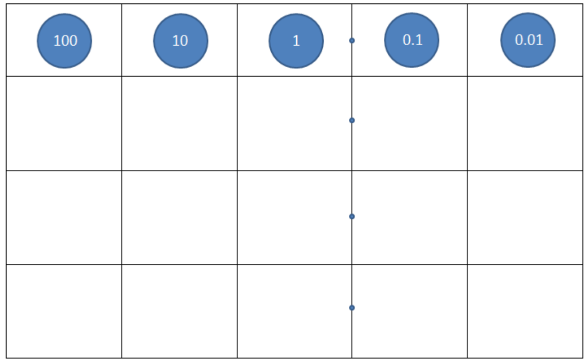
Step 2:	Continue practicing above skills. Count in steps of 2, 3 and 5, forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy, £ and p bank as well as a number square.	Bridging through two digit numbers, i.e. 24 – 19 = 19 + 1 + 4 using number lines. Subtracting 11 by subtracting 10 and then 1 more. Move to subtracting 9 by subtracting 10 and adding 1 using apparatus.	Recall subtraction (and addition) facts for all numbers to 20.	**Subtracting by counting back and on:** • Children begin to use numbered lines to support their own calculations, initially counting back in ones before beginning to work more efficiently. • **Pupils are taught to choose whether to count on or back.** Towards a written method: Using a place value mat and apparatus to support understanding of place value and to prepare for a formal written method.	Number line with all numbers labelled  13 – 5 = 8  And counting on…  13 – 5 = 8   75-42= 

Step 3:	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on and back by 10 or 100 from any two digit number. Link to counting stick counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Partitioning by bridging through 10 and multiples of 10 when subtracting. Continue to practice adjusting when subtracting 11 or 9 from a number. Relating inverse number operations – use structured apparatus to explore and understand that subtraction undoes addition.	Connect subtractions from ten to subtractions from multiples of 10 totalling 100.  Use 10ps in tens frame. Subtract two digit numbers from 100 i.e. ? = 100 - 78	**Finding the difference:** • Teachers model how to find the difference when two numbers are relatively 'close together.' • Initially children compare two sets before moving on to a number line comparison. • **Pupils are taught to choose whether to count on or back.** **Subtracting TU – U and TU – TU:**	Comparing two sets: comparison or difference.  Finding the difference on a number line.  Note: Finding the difference is often the most efficient way of solving a subtraction problem, e.g. 61 – 59 2,003 – 1,997 Introduction to expanded column subtraction: 98-35= 

					For some children this will lead to exchanging, modelled using place value counters (or Dienes). 72-47= 
Step 4:	Continue practicing of previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 11 - 3 = 8, I also know 1.1 - 0.3 = 0.8 and 8/100 - 3/100 = 5/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Subtraction of fractions totalling 1, i.e. 1 – 0.3 = 0.7	**Subtracting TU – U and TU – TU:** The concept of exchange is introduced through continued use of practical equipment (manipulatives). Teachers model: 1. "I have seven tens and two ones. I need to subtract four tens and seven ones." 2. "At the moment, I cannot subtract seven ones from two ones, so I need to transfer one ten to become ten ones." 3. "Now I can take away seven ones from	Use empty number lines to find the difference by bridging through multiples of ten. Subtract by starting with the first number and partitioning the second, i.e. 74 - 27 74 – 20 = 54 54 – 4 = 50 50 – 3 = 47 Formal columnar: 

				twelve ones, so that I have fives ones left. 4. “I can now subtract four tens from six tens, which leaves me with two tens.” 5. “I recombine two tens and fives ones to understand that I am left with twenty-five.”	   Children should continue to use empty number lines and use more formal written methods when numbers become too big or complex. 47 – 23 = 24 

Step 5 - 6:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages. Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Use apparatus and knowledge of place value to subtract decimals, i.e. 3.8 - 2.5 = 1.3 Reorder increasingly complex calculations, i.e. 1.7 – 5 – 0.7 = 1.7 – 0.7 – 5. Compensating – i.e. 405 - 399 → subtract 400 and then add 1. Bridging through decimals, i.e. 1.5 – 0.8 = 1.5 – 0.5 then -0.3 using empty number line.	Continue to practice previous Step and make links between known facts and addition pairs for fractions, percentages and decimals. Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7 Ensure all children are confident recalling basic facts to 20 and deriving using place value. Make links between decimals, fractions and percentages.	Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete materials, e.g. straws, Numicon, Dienes and place-value cards. Teachers similarly advance to model the subtraction of one 3-digit number from another, e.g.	Formal method of decomposition, initially modelled with place value counters  This could also be done using dienes.  Teachers similarly advance to model the subtraction of one 3-digit number from another, e.g. 5 1 5~~6~~3 246 317

				Children should be introduced to written subtraction of decimals	 34.67 <u>21.45-</u> 13.22

Multiplication:

Mental Calculation Strategies for Multiplication and Division

Knowing multiplication and division facts to 12 X 12

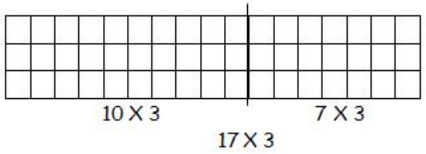
Multiplying and dividing by multiples of 10

thousands	hundreds	tens	ones

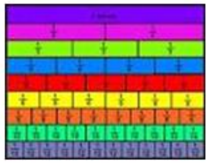
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

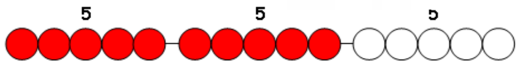
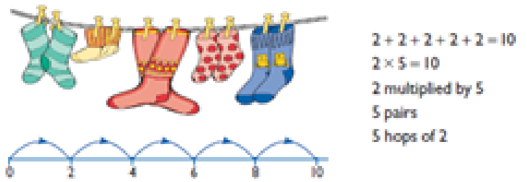
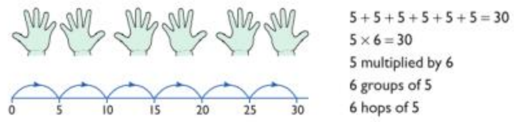
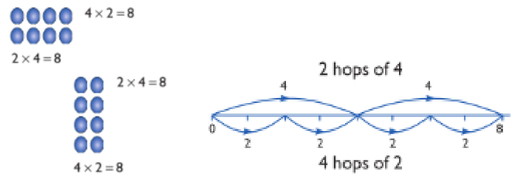
Doubling and halving

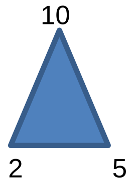
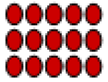
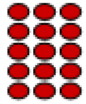
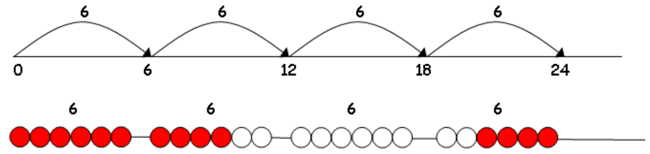
Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers

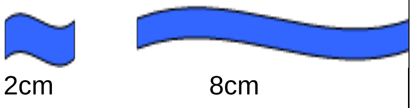
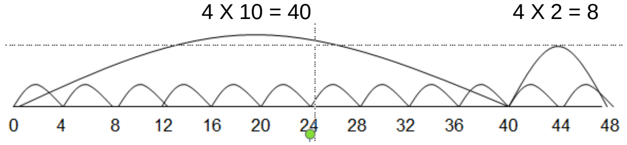
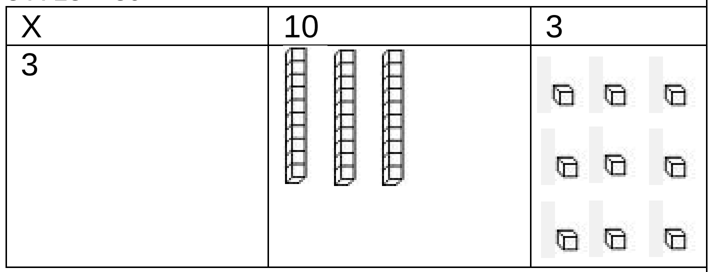
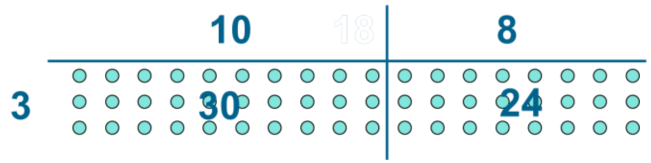


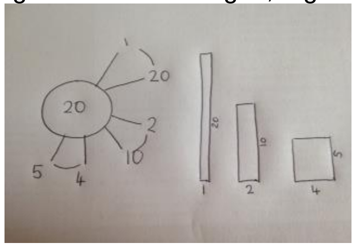
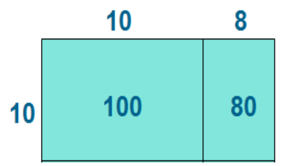
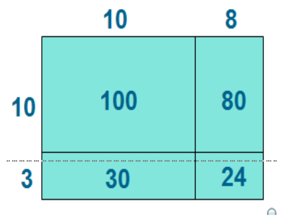
Finding fractions, decimals and percentages

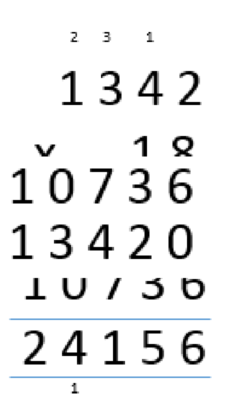
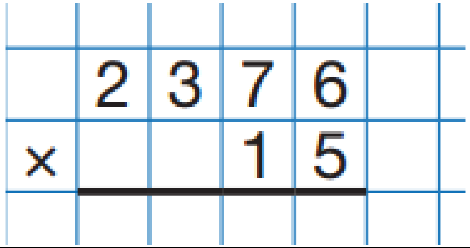


	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Step 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten. Recall odd and even numbers to 10 in reference to structured apparatus. 	**Developing early conceptual understanding of multiplication:**	Use objects, pictorial representations and arrays to show the concept of multiplication:     Using arrays to understand multiplication can be done in any order.  3 6 9 12 15 18 21 24 27 30 

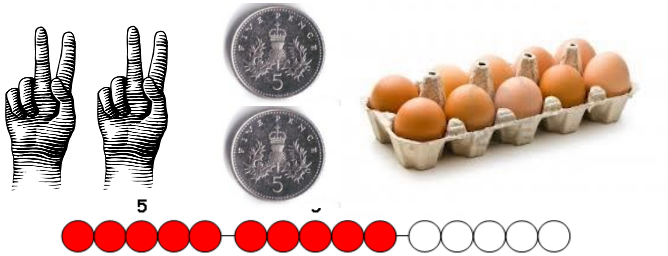
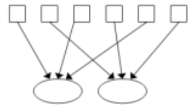
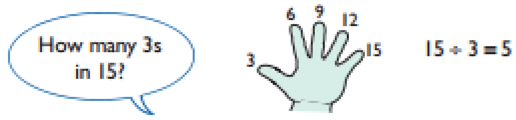
Step 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations:  Stories are used alongside a triad to help children understand links between number operations, e.g. “There are five pencils in two packs, which means that there are ten pencils altogether.”	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall odd and even numbers to 20 in reference to structured apparatus. Recall & use multiplication facts for the 2X, 5X and 10X-tables.	**Understanding multiplication as repeated addition:** • Investigate multiplication as repeated addition, so that the law of cummutativity is understood. • Whilst arrays are also modelled explicitly at this step, it is important to note that they will continue to be a key model at later Steps, alongside more formal methods of calculation.	Arrays: 5 X 3  and 3 X 5  Number lines: 4 X 6 = 24  So: ‘Six taken four times” 3x5= Repeated addition: 5+5+5=  Is the same as: 3x5=

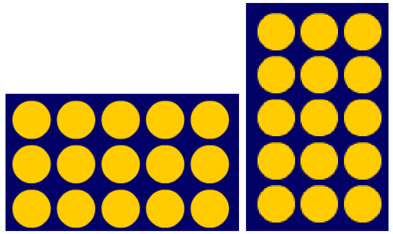
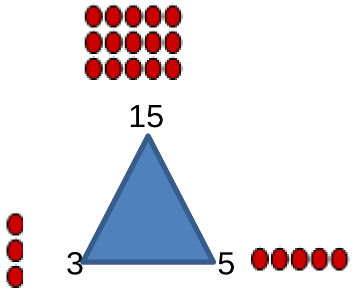
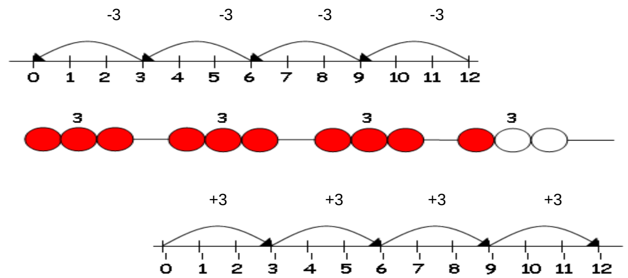
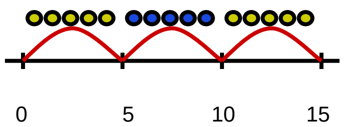
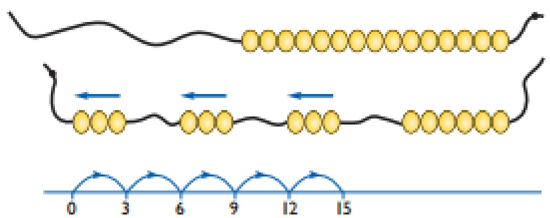
Step 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero. Count up and down in tenths.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon 	Recall odd and even numbers to 100 in reference to structured apparatus. Recall and use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	**Relate multiplying a 2-digit by 1-digit number using repeated addition and arrays to represent:** Written methods (progressing to 2d x 1d) Developing written methods using understanding of visual images Develop onto the grid method	Children use an empty number line to chunk efficiently: 4 X 12 = 48  3 X 13 = 39   3x18=54 

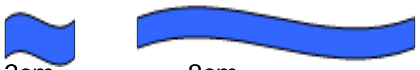
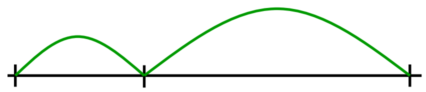
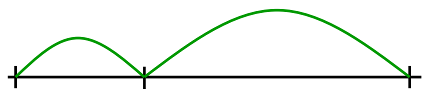
Step 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero. Count up and down in tenths and hundredths.	Derive factor pairs of numbers using models and images, e.g.  Know what happens when a number is multiplied by zero or one. Use reordering to multiply three numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 4/3/2-digit by 2-digit number with grid towards using long/short multiplication:**	 100 80+ 180
Step 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using short multiplication:**	 18 X13 54 2 180 234
Step 6:	Consolidate all previous counting, including forwards and	Perform mental calculations, including with mixed numbers and operations.	Recall & use multiplication facts for all times-tables up to 12 X 12. In	**Relate multiplying a 4/3/2-digit by 2-digit number with grid to using short multiplication:**	

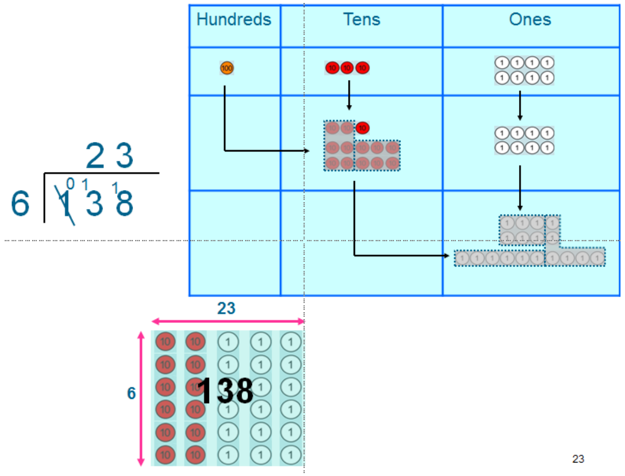
	backwards in fractions.		addition, use facts confidently to make larger calculations.		 Example from SATs paper: 

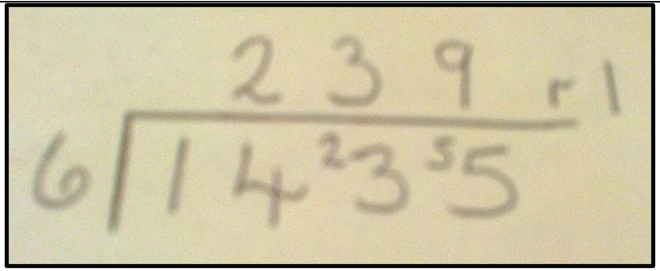
Division:

	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Step 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten. Recall odd and even numbers to 10 in reference to structured apparatus. 	**Developing early conceptual understanding of division as grouping and sharing:**	Use objects, pictorial representations and arrays to show the concept of division as grouping and sharing.  "Two children share six pencils between them"  "Six children are asked to get into three equal groups"  **<u>Grouping</u>** Children should apply their counting skills to develop some understanding of grouping. 

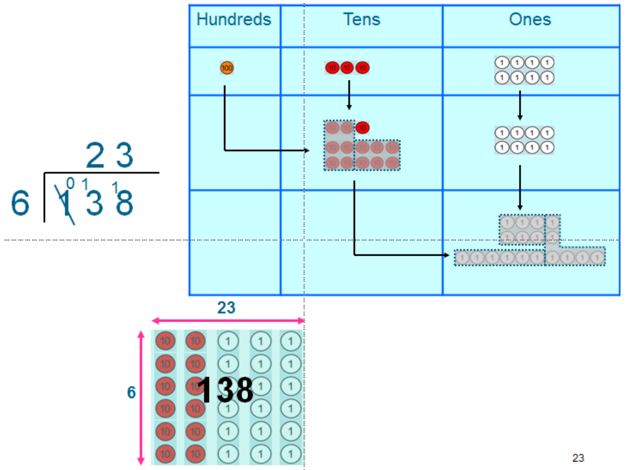
					Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3. 15 ÷ 5 = 3 There are 3 groups of 5.  Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.
Step 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations.  Stories are used alongside a triad to help children understand links between number operations, e.g. “15 children are asked to get into three groups and find out that there are five people in each group.”	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall odd and even numbers to 20 in reference to structured apparatus. Recall and use multiplication facts for the 2X, 5X and 10X-tables.	**Understanding division as repeated subtraction:** • Investigate division as repeated subtraction. • Through teacher modelling, children need to know that division is not commutative. **Grouping using a numberline** Group from zero in jumps of the divisor to find our ‘how many groups of 3 are there in 15?’. 15 ÷ 3 = 5	Number lines and arrays: 12 ÷ 3 = 4 (Show with 12/3)  15 ÷ 5 = 3  

					9÷3= 
Step 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon.  2cm 8cm	Recall odd and even numbers to 100 in reference to structured apparatus. Recall & use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	**Dividing a 2-digit by 1-digit number, representing this efficiently on a number line:**	Children use an empty number line to chunk efficiently. 96 ÷ 6 = 16 (show with 96/6) 6 x 6 = 36 10 x 6 = 60 
Step 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.	Derive factor pairs of numbers using models and images. Know what happens when a number is multiplied by zero or one. Use reordering to multiply three numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 3/2-digit by 1-digit number, representing this efficiently on a number line, also in relation to short division:** At this Step, no remainders are present unless in a practical context.	Children use an empty number line to chunk efficiently. 224 ÷ 8 = 28 (show with 224/8) 8 x 8 = 64 20 x 8 = 160 

				The language of **groups** in relation to the practical method at this stage is vital. It helps children to understand the process of exchange. e.g. Looking at the hundreds, how many groups of 6 can we make? None. So we exchange. Now we have 13 tens. How many groups of 6 can we now make? 2. Exchange the leftover ten for ten ones. How many groups of 6 ones can we make? 3.	Short division: 138 ÷ 6 = 23 
Step 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 1-digit number, in relation to short division:**	<u>**Formal Written Methods**</u> E.g. 1435 ÷ 6

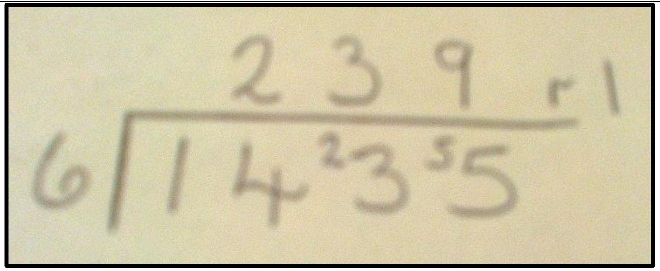


Short division:
138 ÷ 6 = 23



23

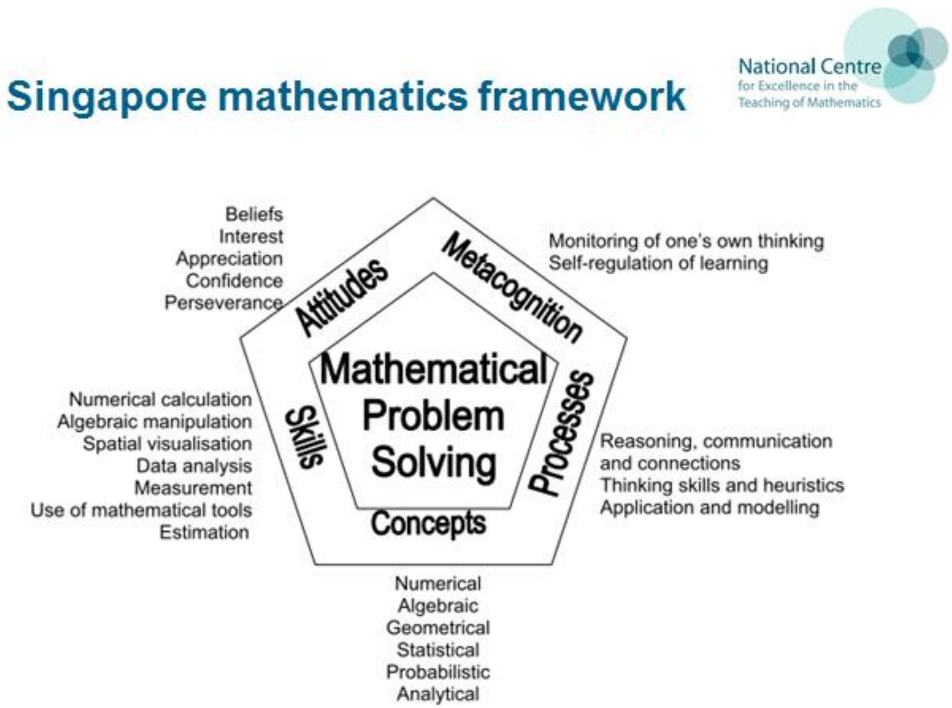
Short division may begin to be taught alongside long division, but still with use of visual representations.

					
Step 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and different number operations.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 2/1-digit number, in relation to long division:** **Be able to write remainders as a fraction and to two decimal places**	Long division: 432 ÷ 15 = 28 4/5 2 8 1 5 | 4 3 2 3 0 0 15×20 1 3 2 1 2 0 15×8 1 2 $\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$ Answer: 28 $\frac{4}{5}$ And 28.80

Appendix 1:

Progression in Use of the Bar Model – Written by the Oxfordshire Maths Team – July 2014

The bar model was introduced in Singapore in the 1980s alongside an increased emphasis on problem solving in the curriculum. A new curriculum was devised based heavily on the Cockcroft report (1982) and ‘Agenda in Action’ (1989) – with the latter being an American report. It fits a maths mastery model, which involves spending more time on a topic. This allows students opportunities to explore maths in-depth. Such a concept has also been stressed by Jane Jones; lead HMI for mathematics, when she talks about ‘milking the maths.’

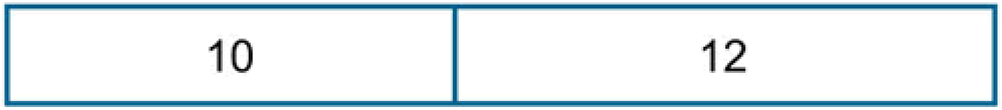


The Singapore and Shanghai curriculums have problem solving at the heart of them. Arguably, this is why they are so successful. This model also reflects strong teaching in this country, as noted in OfSTED’s ‘Made to Measure’ report. (2012)

The purpose of this document is to illustrate progression within the bar model and is the Oxfordshire maths team’s attempt to show where it could fit within the new national curriculum. It could become part of a school’s calculation policy. Please note that use of the bar model is not statutory and is designed to help children visualise maths problems, rather than do the maths for them. We have drawn heavily on various NCETM resources in order to write this document.

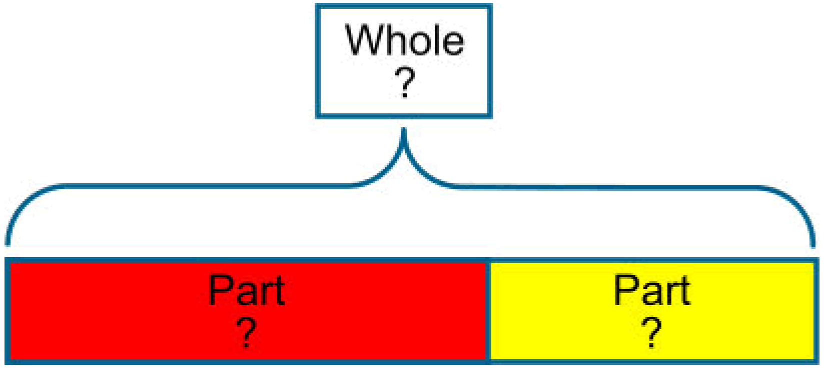
What does the bar model look like?

Sam had 10 red marbles and 12 blue marbles. How many marbles did he have altogether?



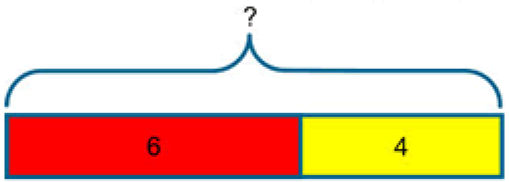
$10 + 12 = 22$

In problems involving addition and subtraction there are three possible unknowns as illustrated below and given the value of two of them the third can be found.



The examples below illustrate a variety of ways that the bar might be used for addition and subtraction problems. A question mark is used to indicate the part that is unknown.

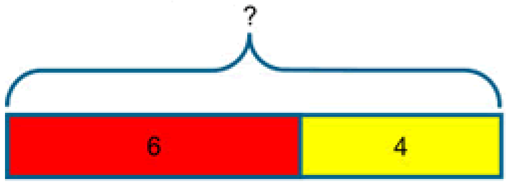
Addition
Aggregation
- two quantities combined



I have 6 red pencils and 4 yellow pencils. How many pencils do I have?

(I combine two quantities to form the whole)

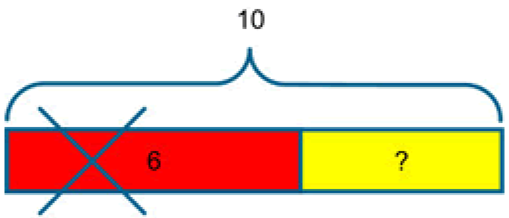
Addition
Augmentation
- a quantity is increased



I have 6 red pencils and I buy 4 yellow pencils. How many pencils do I have?

(The bar I started with increases in length)

Subtraction
- Take Away



I had 10 pencils and I gave 6 away, how many do I have now?

(This time we know the whole but only one of the parts, so the whole is partitioned and one of the parts removed to identify the missing part)

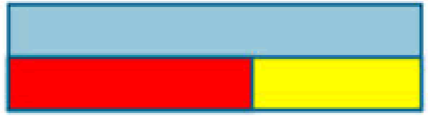
Subtraction
- Comparison or Difference

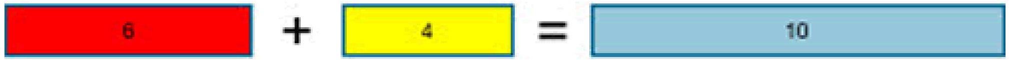
Tom has 10 pencils and Sam has 6 pencils. How many more does Tom have?

(The bar is particularly valuable for seeing the difference between the two quantities)

Equivalence

The model can be rearranged to demonstrate equivalence in a traditional layout

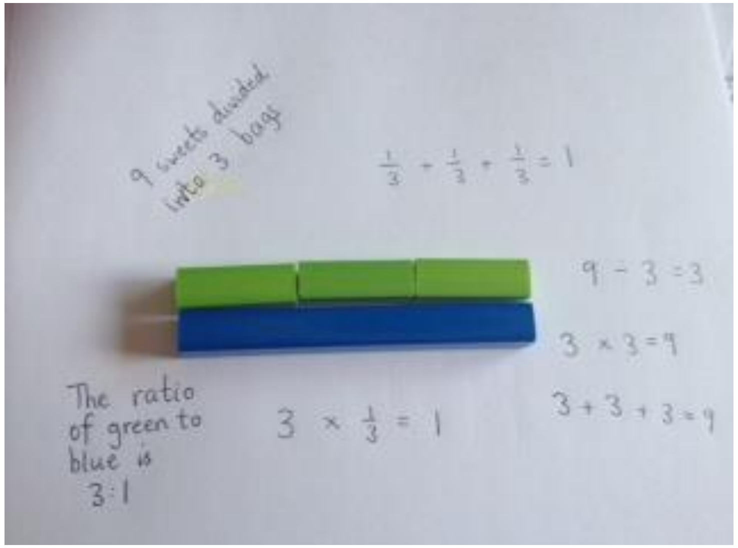




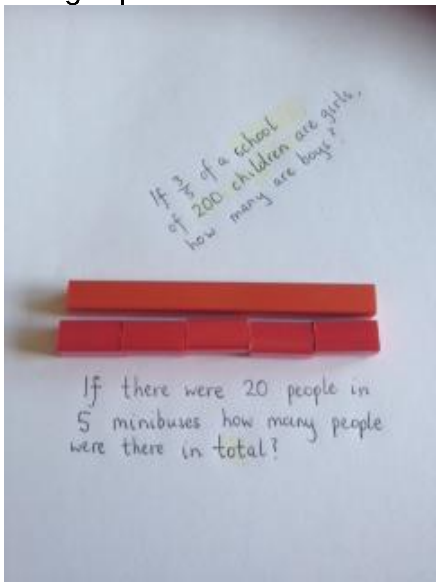
Pupils need to develop fluency in using this structure to represent addition and subtraction problems in a variety of contexts using the bar model. The model will help children to see that different problems share the same mathematical structure and can be visualised in the same way. Asking children to write their own problems, using the bar as the structure will help to consolidate this understanding.

Milking the maths: using the bar model flexibly across all year groups:

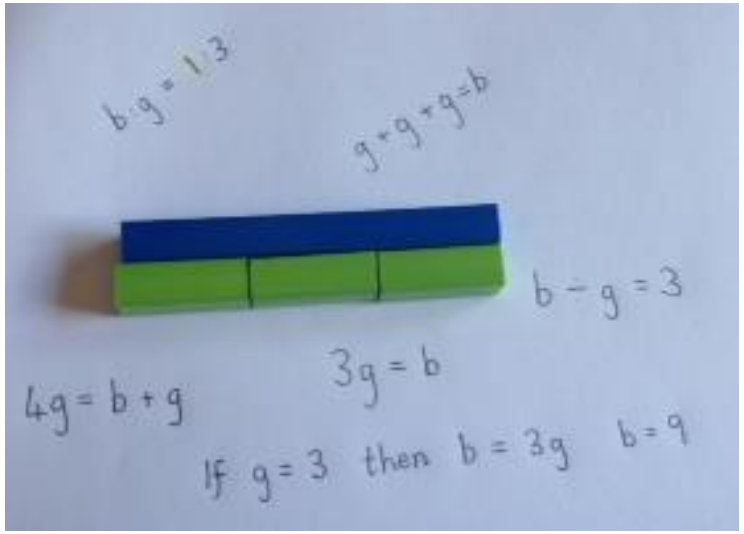
What could this bar model be showing?



Can you design questions that fit the bar model?



Playing with algebra



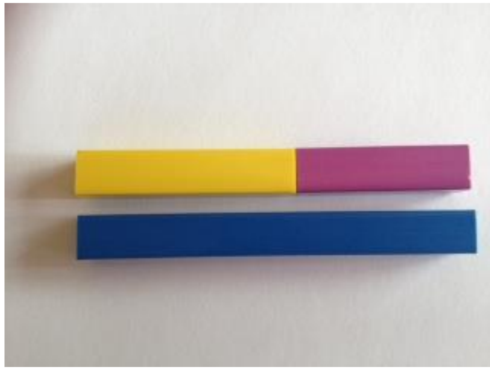
The maths team feel that Cuisenaire should be used by children at this stage, so that the gap is bridged between using concrete objects and drawing symbolic representations of objects. We would expect children to be working practically with Cuisenaire and talking about relations, so that they are then ready to start drawing bar model representations in lower Key-Stage 2.

National Curriculum Objectives: Rapid Recall

Year 1: represent and use number bonds and related subtraction facts within 20
Year 2: recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

National Curriculum Objectives: Addition and Subtraction

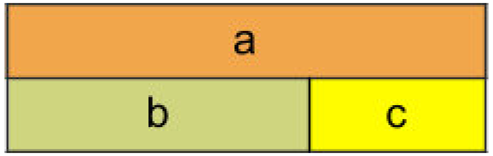
Year 2: show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot



Focus on verbalising thinking:
e.g.
'five add four is equal to nine'
'four add five is equal to nine'
'nine take-away four equals five'
'nine take away five equals four'

Addition and Subtraction

The bar model supports understanding of the relationship between addition and subtraction in that both can be seen within the one representation and viewed as different ways of looking at the same relationships.



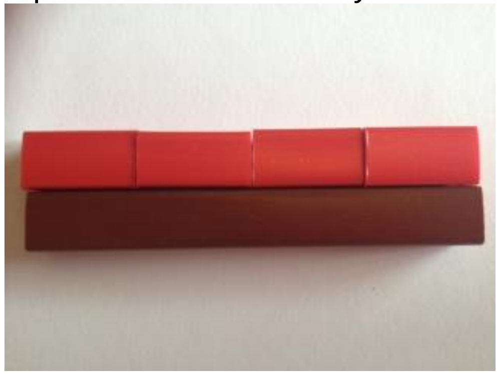
This diagram encapsulates all of the following relationships;

$a = b + c$; $a + c + b$; $a – b = c$; $a – c = b$

For teacher's understanding, children would not be expected to write the above algebraic notations.

National Curriculum Objectives: Multiplication and Division

Year 1: solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.



Focus on verbalising thinking:
e.g.
'eight is two taken four times'
'two taken four times is eight'
'eight equals four times two'
'there are four twos in eight'

Year 2: solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

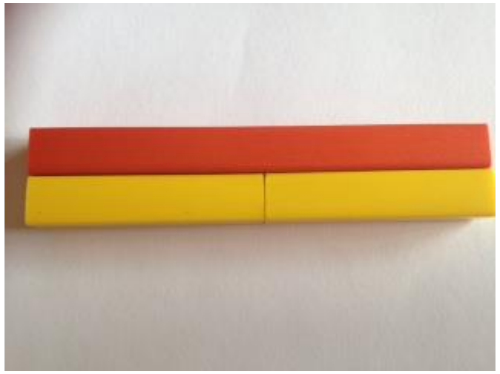
[Photograph of Cuisenaire being used as a bar model and arrays.]

National Curriculum Objectives: Fractions

Year 1: recognise, find and name a half as one of two equal parts of an object, shape or quantity
recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

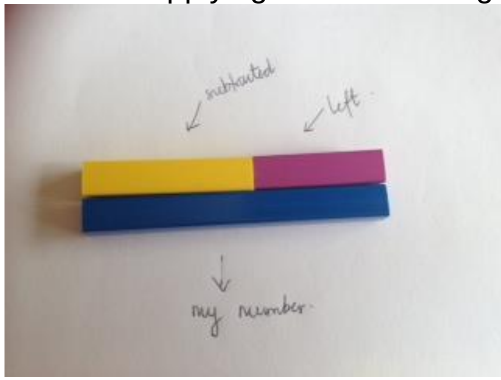
Year 2: recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity

write simple fractions e.g. $^1/_2$ of 6 = 3 and recognise the equivalence of $^2/_4$ and $^1/_2$.



e.g.
What fractions can you see?
What fraction of the orange is each yellow piece?
If the value of the orange rod is ten, what is the value of each yellow rod?
Extension: What if the value of the orange rod is 6? 100? Etc.

National Curriculum Objectives: Problem Solving
Year 1: solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9. Year 2: solve problems with addition and subtraction: ○ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ○ applying their increasing knowledge of mental and written methods



e.g.
I think of a number. I subtract 5. The answer is 4. What is my number?

e.g.
A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?

Courtesy of the NCETM:
These problems have been tried and tested in a school for the year groups specified; however you may wish to use them more flexibly for different aged children than that specified. Please note that schools can decide in which year group to use each of the problems, as some are challenging.

Year 1 Problems

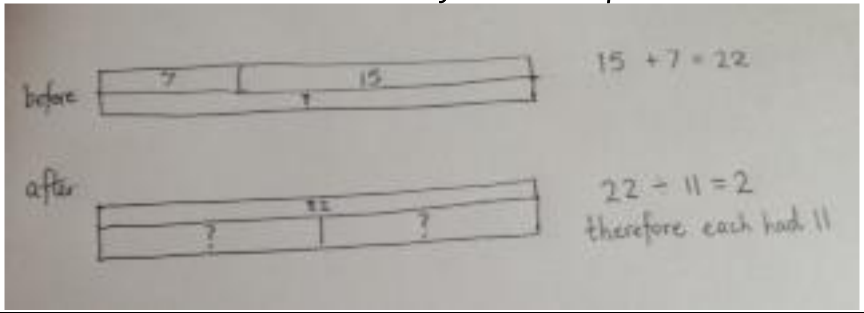
1. Ebony has 5p and Daniel has 8p. How much do they have altogether?
2. A lolly costs 6p. Amrit paid with a 10p coin. How much change does he get?
3. Michael says that 16 + 5 = 21. Is he correct?
4. I think of a number. I subtract 5. The answer is 4. What was my number?
5. How many gloves are there altogether in 6 pairs of gloves?
6. Twelve people are split into two groups. How any are in each group?
7. Mrs Morton puts five 5p coins into her purse. How much is in her purse altogether?

Year 2 Problems

1. Dylan has 37 coloured pencils and he buys 30 more. How many does he have now?
2. Janie has 40 beads. She loses 25 of them. How many does she have left?
3. What is the difference between seventy six and thirty five?
4. I think of a number. I subtract 5. The answer is 4. What was my number?
5. Last week Ellie got £1.00 pocket money. She spent half of it. How much has she got left?
6. A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?
7. Amelia writes the calculation below as a multiplication calculation? What might she write?
3 + 3 + 3 + 3 + 3 = 15
8. Mr Siddique shares £18 equally between his three sons. How much does each son get?
9. Charlotte-May had to find a 14 of a number. Her answer was 4. What number did she start with?
10. Danny cuts his pizza into 8 equal slices. He eats 3/4 of the pizza and gives the rest to his dog, Gruff. How many pieces does Danny eat?

Years 3 and 4:

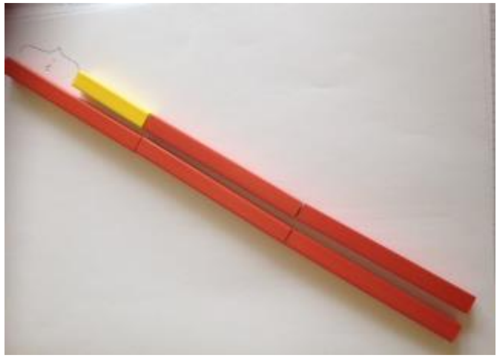
The maths team feel that children should now be ready to start drawing bar model representations. For the purpose of this progression document, we have modelled 'before' and 'after' representations of problems. It may, however, be that schools instead choose to use a model whereby the first representation is developed.



e.g.
Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?

National Curriculum Objectives: Problem Solving in Addition and Subtraction

Year 3: solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
Year 4: solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.



e.g. one-step problem
Sally has 40 football cards. She gives 25 of them away. How many does she give away?

National Curriculum Objectives: Problem Solving in Multiplication and Division

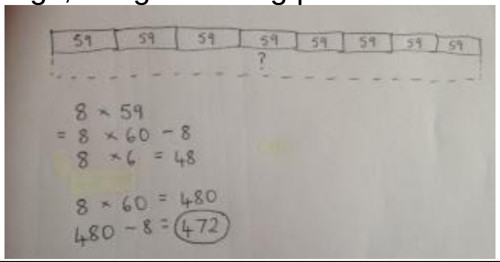
Year 3: solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects.

Peter has 4 books
Harry has five times as many books as Peter.
How many books has Harry?

This is an example of integer scaling.

4 × 5 = 20
Harry has 20 books

Year 4: solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.



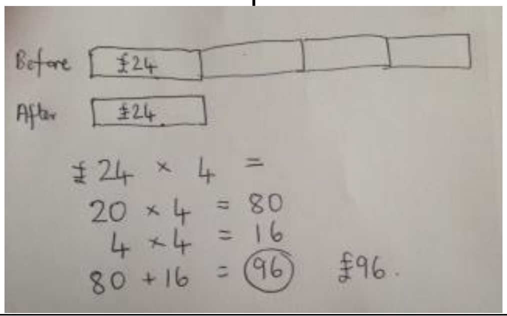
e.g. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?

National Curriculum Objectives: Fractions

Year 3: solve problems that involve (unit fractions and fractions with the same denominator)

Year 4: solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number

Year 4: solve simple measure and money problems involving fractions and decimals to two decimal places, including decimals



e.g. A computer game is £24 in the sale. This is one quarter of its original price. How much did it cost before the sale?

Courtesy of the NCETM:
These problems have been tried and tested in a school for the year groups specified; however you may wish to use them more flexibly for different aged children than that specified.

Year 3 Problems

1. There are 334 children at Springfield School and 75 at Holy Trinity Nursery. How many children are there altogether?
2. Gemma collected 293 badges but she gave 45 of them to her friend, Rebecca. How many badges did she have left?
3. Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?
4. Seven people each put five pens into a pot. Carmen then takes out fifteen pens. How many pens are left?
5. If you spend 61p at the corner shop, how much change do you get from £1.00?
6. If five apples cost fifty pence, how much would two apples cost?
7. Emma buys seven markers for 30p each. How much change does she get from £3.00?
8. A bookcase in the library holds 5 shelves with 46 books on each shelf. How many books are there in the bookcase altogether?
9. How many 5p stickers can Alexis buy with his 55p pocket money?
10. Which is the larger amount, one third of £60 or one quarter of £88?
11. A computer game is £24 in the sale. This is one quarter of its original price. How much did it cost before the sale?

Year 4 Problems

1. Martin has saved £6.78 and spends £4.69. How much does he have left?
2. Sally has 40 football cards. She gives 25 of them away. How many does she give away?
3. Sally has 30 football cards. She gives 25 of them to her friend. How many does she have left?
4. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?
5. Calculate how many fives there are in 85?
6. At the dressmakers, Debbie buys buttons weighing 3 grams each. If she has 81 grams of buttons, how many buttons does she buy?
7. Kelly buys four fifths of the shop’s oranges. If the shop had 20 oranges, how many does she have?

Years 5 and 6:

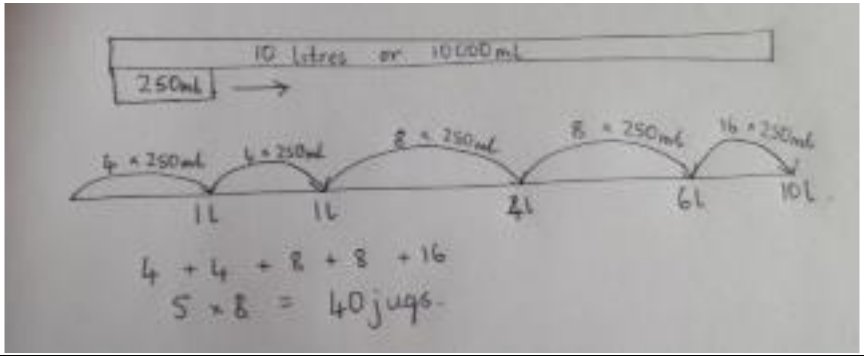
The maths team feel that Cuisenaire and double-sided counters are still valuable resources to use alongside drawn representations of problems. They should all be used flexibly according to the stage that children have reached in their learning.

National Curriculum Objectives: Problem Solving in Addition and Subtraction

Year 5: solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
Year 6: solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

National Curriculum Objectives: Problem Solving in Multiplication and Division

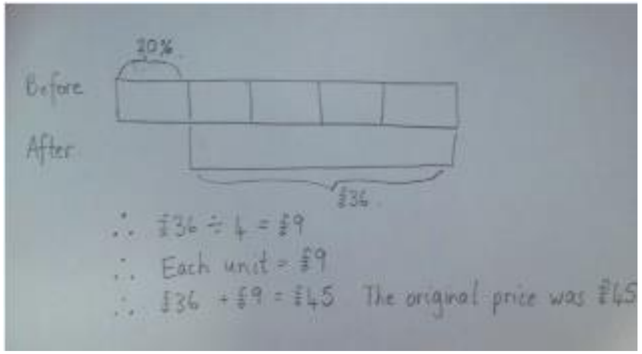
Year 5: solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates



e.g. How many jugs with a capacity of 250ml could you fill with 10 litres of water?

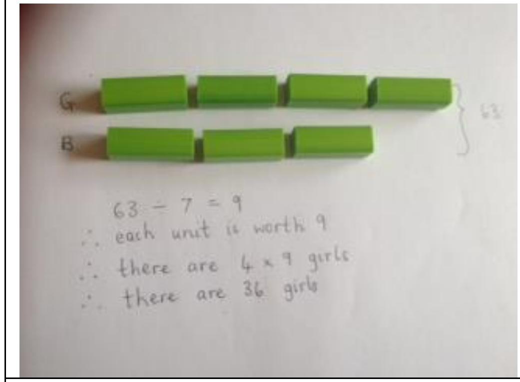
National Curriculum Objectives: Fractions

Year 5: solve problems which require knowing percentage and decimal equivalents of $^{1}/_{2}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{4}/_{5}$ and those with a denominator of a multiple of 10 or 25, including decimals and percentages
Year 6: solve problems which require answers to be rounded to specified degrees of accuracy, including decimals and percentages



e.g. There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?

The bar model can also be linked beautifully with ratio and proportion, for example, in the following context:



e.g. At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?

Courtesy of the NCETM:

These problems have been tried and tested in a school for the year groups specified; however you may wish to use them more flexibly for different aged children than that specified.

Year 5 Problems

- 1. Every day for 4 days Helen scored 7.5 in a test. On the fifth day she scored 8. What was her total score?
- 2. I cut 60 cm from 3.3m of string and shared the rest between 3 friends. How much string did they get each?
- 3. How many jugs with a capacity of 250ml could you fill with 10 litres of water?
- 4. All the children in the school are going on a residential trip to the outdoor activity centre. They will be divided into 6 equal groups If there are 246 children in the school how many will be in each group?
- 5. Robert calculated 25% of 600. What answer does he get?
- 6. Sam calculated 40% of 120. What answer does he get?
- 7. Rita worked out that 16 of a number was 12. What was the number she started with?

Year 6 Problems

- 1. Three quarters of a number is 54. What is the number?
- 2. Which is more; 59 of 252 or 47 of 238?
- 3. There are 36 packets of biscuits. One half are chocolate, a ninth are digestive and a third are wafer biscuits. The rest are ginger nuts. How many biscuits are ginger nuts?
- 4. There is 20% off in a sale. How much would a track suit cost, if the normal price was £44.50?
- 5. There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?
- 6. At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?
- 7. Seven in every nine packets of crisps in a box are salt and vinegar. The rest are plain. There are 63 packets of salt and vinegar crisps. How many packets of plain crisps are there?

Key Stage 3 Problems

1. Ralph posts 40 letters, some of which are first class, and some of which are second class.
He posts four times as many second class letters as first.
How many of each class of letter does he post? (This question appeared on a GCSE higher tier paper.)
2. A computer game was reduced in a sale by 20% and it now costs £55. What was the original cost?
3. A computer game was reduced in a sale by 30% and it now costs £77. What was the original cost?
4. Sally had a bag of marbles. She gave one-third of them to Rebecca, and then one quarter of the remaining marbles to John.
Sally then had 24 marbles left in the bag. How many marbles were in the bag to start with?
5. Peter is playing Space Explorer on his computer. He finished 13 of the levels last week and 25 of the remaining levels this week.
He has 12 more levels to complete. How many levels does Space Explorer have?
6. Sam bakes a variety of biscuits.
13 are peanut, 12 are raisin, the remaining 12 were oat. How many biscuits are baked?
7. Tom spent 30% of his pocket money and put away 45% into his savings. He was left with £2.50. How much pocket money did he receive?
8. Two numbers are in the ratio 4:5. They both sum to 135. Identify both numbers.
9. Two numbers are in the ratio 6:7. They both sum to 169. Identify both numbers.
10. A herbal skin treatment uses yoghurt and honey in the ratio 5:3. How much yoghurt is needed to mix with 130 g of yoghurt?